Orientation-Dependent Entanglement Lifetime in a Squeezed Atomic Clock

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We study experimentally the lifetime of a special class of entangled states in an atomic clock, squeezed spin states. In the presence of anisotropic noise, their lifetime is strongly dependent on squeezing orientation. We measure the Allan deviation spectrum of a clock operated with a phase-squeezed input state. For integration times up to 50 s the squeezed clock achieves a given precision 2.8(3) times faster than a clock operating at the standard quantum limit.

Atomic interference provides an exquisitely sensitive tool for measuring gravitation, magnetic fields, acceleration, rotation, and time itself [1, 2]. It has long been hoped that quantum-mechanical entanglement might enhance the precision of such measurements: maximally-entangled states can increase the sensitivity of the interference fringe to the parameter of interest [3], while squeezed spin states can redistribute quantum noise away from that quantity [4, 5]. In experiments, both approaches have overcome the standard quantum limit (SQL) of phase sensitivity [6–11]. However, Huelga et al. pointed out early on that entangled states might provide little gain in real metrological performance because they are more fragile than uncorrelated states, such that the entanglement-induced increase in phase sensitivity comes at the expense of reduced interrogation time [12]. Analyses with specific noise models [13, 14], however, found parameter regimes where entanglement could be helpful despite decoherence. It is thus interesting, practically as well as fundamentally, to investigate the fragility of the entangled states relevant to metrology.

In this Letter we show that for an atomic clock in which the dominant environmental perturbation is phase noise, the squeezed-state lifetime varies by an order of magnitude depending on whether the squeezed variable is the phase (subject to environmental perturbation) or the (essentially unperturbed) population difference between states. We operate an atomic clock with a phase-squeezed input state whose precision exceeds the SQL, as also recently demonstrated by Louchet-Chauvet et al. [11], and present the first measurement of such a clock's Allan deviation spectrum. The clock reaches a given precision 2.8(3) times faster than the SQL for integration times up to 50 s. The squeezed states used in this work are prepared by cavity feedback squeezing [10, 15], a new technique which deterministically produces entangled states of distant atoms using their collective interaction with a driven optical resonator.

Given any two-level atom we can define a spin-1/2 $\boldsymbol{s}_i$. For an ensemble of such atoms, we introduce the total spin $\boldsymbol{S} = \sum \boldsymbol{s}_i$ whose $\hat{\boldsymbol{z}}$ component and azimuthal angle ϕ represent the population difference and relative phase, respectively, between the two atomic levels. Simultaneously preparing the atoms in the same single-particle

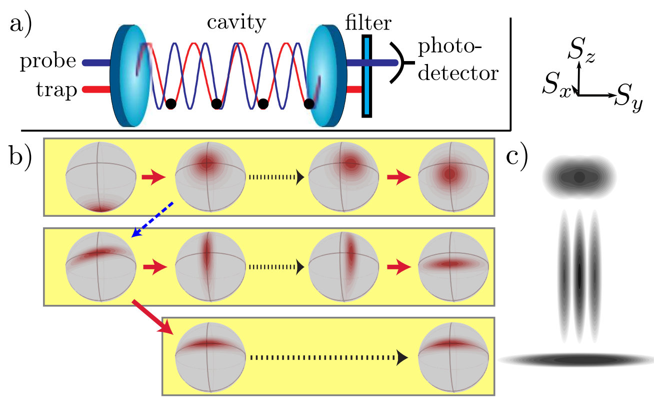

FIG. 1. **(a) Setup:** A standing-wave dipole potential confines ^{87}Rb atoms inside an optical resonator where they interact with probe light via their S_z-dependent index of refraction. The probe light is used for cavity feedback squeezing [10, 15] and final-state readout. **(b) Pulse Sequences:** A standard Ramsey protocol (top bar) consists of optical pumping into the spin-down CSS, rotation into the equatorial plane using a microwave pulse (red arrow), free precession time (dotted black arrow), and conversion of the accrued phase into a population difference for readout. We can shear the CSS into a squeezed state using cavity feedback (dashed blue arrow), then orient the narrow axis of the squeezed state in the phase direction (middle bar) or in the $\hat{\boldsymbol{z}}$ direction (bottom bar). **(c) Effect of Phase Noise:** At the end of the free precession time, classical uncertainty on the accrued phase broadens the CSS in one direction (top). The same noise is detrimental to phase-squeezed states (middle) but much less noticeable in number-squeezed states (bottom) whose narrow direction is not affected.

quantum state places the ensemble in a coherent spin state (CSS) where the variance of the spin components perpendicular to the mean spin is given by $|\langle \boldsymbol{S} \rangle|/2$ [16]. While loss of coherence between the atoms can shorten $|\langle \boldsymbol{S} \rangle|$ and reduce this variance, only entanglement between atoms can improve the signal-to-noise ratio (SNR) of a measurement of the orientation of $\boldsymbol{S}$ [4, 5]. For a system with $2S_0$ total atoms and signal contrast C (C_{in}) for the correlated (uncorrelated) state after (before) the squeezing procedure, we can define a metrological squeezing parameter [4] $\zeta = 2\Delta S_\perp^2 C_{\text{in}}/(S_0 C^2)$, which compares the squared SNR for the best possible measurement on a

CSS with the spin length $C_{\text{in}}S_0$ to that of the actual measurement with transverse variance $\Delta S_\perp^2$ and spin length $|\langle \boldsymbol{S}\rangle| = CS_0$. If $\zeta < 1$, the total spin orientation is more precisely determined, in some direction, than would be possible if the atoms were not entangled. The spin state is then squeezed, and the individual atoms in the ensemble are necessarily entangled [17].

If we prepare a CSS in the equatorial plane, we expect that the phase noise will increase at long times due to classical fluctuations on the energy difference between levels in our apparatus (Fig. 1c, top). As discussed below, this broadening becomes noticeable in our system in less than 1 ms. In the number direction, the primary mechanism that increases polar angle uncertainty $\Delta S_z/|\langle \boldsymbol{S}\rangle|$ is loss of contrast (reduction in the length of the mean spin vector $|\langle \boldsymbol{S}\rangle|$), which occurs on a longer time scale $T_{\text{coh}} = 11(1)$ ms in our apparatus. A phase-squeezed state will therefore suffer much more rapid broadening of its narrow axis (Fig. 1c, middle) than a number-squeezed state (Fig. 1c, bottom). Intriguingly, a number-squeezed state is less vulnerable to phase noise than an uncorrelated state: an increase in phase variance by several times the width of the original CSS might still be small compared to the antisqueezed phase variance of the number-squeezed state. Finding robust orientations for the squeezed state is reminiscent of the search for decoherence-free subspaces in quantum information science [18–20]: with knowledge of the specific noise processes in a particular system, one can find quantum states which are relatively immune to their effects.

We work with laser-cooled ^{87}Rb and use the canonical magnetic-field-insensitive clock transition, choosing $|F = 1, m_F = 0\rangle$ and $|F = 2, m_F = 0\rangle$ as the two clock states. The atomic cloud is held by a dipole trap inside a Fabry-Pérot resonator (Fig. 1a), one mode of which is detuned halfway between the D_2 optical transitions for the two clock states. The total spin $\boldsymbol{S}$ corresponds to a sum over the atomic cloud, weighted by the atoms' position-dependent coupling to the resonator mode so as to yield an effective uniform-coupling description [10]. The index of refraction of the atoms shifts the cavity resonance frequency by equal and opposite amounts for atoms in each of the two clock states, the net shift being proportional to their population difference $2S_z$. In order to read out the atomic state, the resonator is driven by probe light tuned to the slope of the cavity resonance, so that atom-induced shifts of the resonance frequency are revealed as changes in the transmitted fraction of probe light, which we detect on an avalanche photodiode. The probe light is generated as a sideband of a laser locked to another cavity mode, 33 GHz red-detuned from the D_2 transition for the $F = 2$ state in order to reduce the interaction between lock light and atoms. A 2.4 G bias field along the cavity axis combined with circular trap polarization makes the clock frequency first-order independent of trap power [8], at the price of making it linearly sensitive to magnetic

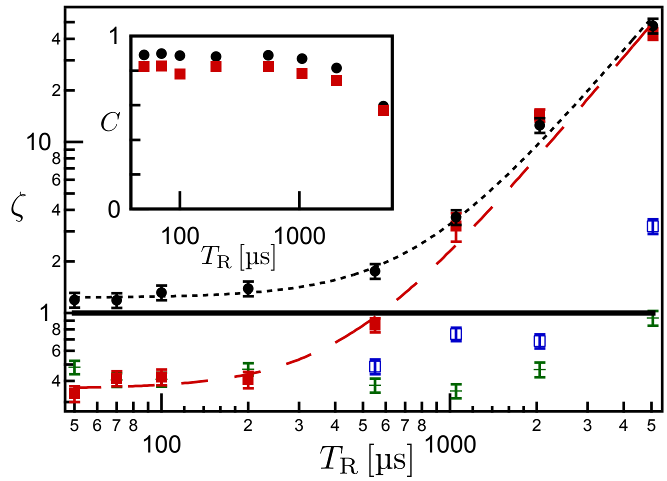

FIG. 2. Metrological squeezing parameter ζ as a function of time. The phase variance for an initial CSS (black circles, dotted line) or phase-squeezed state (red solid squares, dashed line) increases due to classical frequency noise. The number variance of a number-squeezed state (green dashes) or the phase variance of a phase-squeezed state protected by a spin echo (blue open squares) can remain below the projection noise limit (solid black line) substantially longer. Error bars show the statistical uncertainty of the variance determination. Inset: Signal contrast C for the CSS (black circles) and squeezed states (red squares).

field fluctuations with a coefficient of 3.7 kHz/G. These fluctuations are the dominant noise affecting the clock spin. Aside from the choice of lock detuning, bias field, and trap polarization given above, the details of our apparatus and method for preparing squeezed spin states follow the description of Refs. [8, 10].

Arbitrary rotations of the spin vector can be performed using resonant microwave pulses, but squeezing the uncertainty region requires an effective interaction between the atoms which we generate by cavity feedback, exploiting their common coupling to the light field of the resonator [10, 15]. As the atomic index of refraction shifts the cavity resonance by an amount proportional to S_z, it changes the intracavity intensity of probe light. Since the probe imparts a light shift to the atoms, each atom acquires a phase shift which depends on the state of all other atoms in the ensemble, thus introducing the correlations necessary for squeezing. The S_z-dependent phase shift shears the circular uncertainty region of the CSS into an ellipse with its long (antisqueezed) axis oriented at a small and known angle to the equatorial plane (Fig. 1b, middle bar). Note that, due to photon shot noise in the probe light, the states thus prepared are actually mixed states with an area much larger than is required by the Heisenberg uncertainty relations [15].

To measure the lifetime of a phase-squeezed state, we load the dipole trap with an ensemble of atoms collected in a magneto-optical trap (MOT), prepare a CSS by op-

tically pumping the atoms into the $|F=1, m_F=0\rangle$ state and then apply a microwave $\pi/2$ pulse to rotate it into the equatorial plane of the Bloch sphere [10]. We typically work with $2S_0 \approx 3\times 10^4$ effective atoms and an initial contrast $C_{\rm in} = 90(2)\%$, yielding a projection-noise-limited phase uncertainty of $\sim 6\,$mrad [4]. Cavity feedback squeezing with two weak pulses of probe light then drives the atoms into a state with $\zeta^{-1} \approx 4\,$dB. A rotation of nearly $\pi/2$ converts this into a phase-squeezed state. After allowing the spins to precess for a variable time $T_{\rm R}$, the phase information is converted back to a population difference with a final $\pi/2$ pulse and read out by observing the transmission of a pair of strong probe pulses. This sequence of operations constitutes a Ramsey-type atomic clock with a squeezed input state. We perform 10 sequences of state preparation, precession and readout for each sample of atoms loaded from the MOT, and the entire experimental cycle repeats every 9 seconds. We measure the phase variance of the CSS using the same experimental sequence but without the probe light which performs the squeezing. To measure the lifetime of a number-squeezed state, we rotate the sheared state slightly to orient its narrow axis along $\hat{\boldsymbol{z}}$ and then simply hold it for a time $T_{\rm R}$ before reading it out (Fig. 1b, bottom bar). In all three cases, comparing the normalized variance to the squared contrast for the readout signal yields the metrological squeezing parameter ζ, which we plot in Fig. 2. The SQL (black line at $\zeta = 1$) is calculated from independent absolute atom number measurements based on precisely-measured cavity parameters, and verified experimentally as in Ref. [8].

We begin by using the CSS to evaluate the classical phase noise (Fig. 2, black circles). The data are well-described by the model $\zeta(T_{\rm R}) = \zeta(0) + 2S_0 C_{\rm in} \Delta\omega^2 T_{\rm R}^2$ (dotted fit), involving an initial angular uncertainty described by $\zeta(0)$ and additional fluctuations of the transition frequency between measurements with variance $\Delta\omega^2$. After a precession time of $\approx 700\,\mu$s, the effect of the classical noise $\Delta\omega = 2\pi \times 1.3\,$Hz exceeds the initial projection noise and the phase variance increases quadratically thereafter. Note that reaching the SQL in an atomic clock requires not only projection-noise-limited state preparation and readout, but also an interrogation time short enough that quantum projection noise remains the dominant uncertainty on the clock signal; less than $700\,\mu$s in this case.

When we prepare a phase-squeezed state (solid red squares), we initially observe a reduced phase variance $\zeta(0) < 1$, as expected. The same frequency noise $\Delta\omega^2$ broadens the phase-squeezed state somewhat sooner than the CSS because there is less quantum noise to mask the classical fluctuations. Note, however, that the same frequency noise that destroys the squeezed states in $\approx 600\,\mu$s also degrades the performance of initially unsqueezed states, so that states that were initially squeezed provide slightly better SNR even after $\zeta(t)$ increases beyond unity.

Matters are substantially different when we prepare a number-squeezed state and read out its reduced $\hat{\boldsymbol{z}}$-variance directly after a hold time $T_{\rm R}$ (Fig. 2, green dashes). In this case we are not operating a clock, which measures the evolution of the phase angle, but instead examining the evolution of the polar angle corresponding to the population difference between the clock states. Frequency noise does not add uncertainty to this spin component, and so ζ can remain below unity (squeezed) for 5 ms, eight times longer than for the phase-squeezed state, until dephasing between the atoms, visible as loss of signal contrast (Fig. 2, inset), creates a mixed state whose quantum correlations are insufficient to overcome the SQL. There is no measurable change in the phase variance of the number-squeezed state (not shown) out to 5 ms; the classical frequency noise is entirely hidden in the antisqueezed initial phase variance.

Since the decay of the phase-squeezed state results from classical phase noise, standard techniques can suppress it. For example, we have operated a "clock" sequence with a phase-squeezed input state, but with an additional spin-echo pulse halfway through the precession time. The final phase is then insensitive to the atomic transition frequency, which protects the state from slow frequency fluctuations but makes it useless for timekeeping. The state remains squeezed 2 ms after being prepared in the otherwise fragile phase-squeezed orientation (Fig. 2, open blue squares).

Note that several experimentally demonstrated spin squeezing schemes [8–10, 21] prepare the ensemble in or near the robust number-squeezed state. It is therefore possible to prepare squeezed input states for an atomic clock at leisure, while remaining largely insensitive to perturbations of the atomic phase. It is only when the clock is started by a subsequent $\pi/2$ pulse [11] that the squeezed state becomes sensitive to frequency noise.

As a demonstration, we have operated such a clock with a Ramsey interrogation time $T_{\rm R} = 200\,\mu$s, short enough that the classical frequency noise in our system does not destroy the phase squeezing. The effective atom number was $2S_0 = 3.5 \times 10^4$, the clock cycle time was 9 s and the signal contrast was $C = 81\%$. Only a single Ramsey interrogation was performed for each MOT loading cycle, giving a duty factor of 2×10^{-5}. The result is the first measurement of Allan deviation [2] for an atomic clock operating beyond the SQL, including all noise and slow drifts (Fig. 3, red solid line). For comparison, we also evaluate a clock operated with an uncorrelated input state close to a CSS, 100(2)% signal contrast and otherwise identical parameters (Fig. 3, black circles). An ideal projection-noise-limited clock with the same atom number, interrogation time and duty factor could reach a stability $\sigma(\tau) = 1.85 \times 10^{-9}\,{\rm s}^{1/2}/\sqrt{\tau}$ (Fig. 3, dashed black line). At short times our squeezed-state clock reaches a fractional frequency uncertainty of

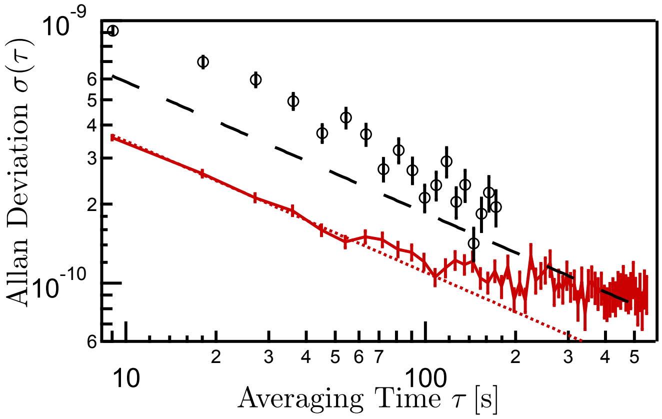

FIG. 3. Allan deviation of a squeezed clock. The solid red line with error bars was measured using a squeezed input state. The dotted red line indicates $\sigma(\tau) = 1.1 \times 10^{-9}\,\mathrm{s}^{1/2}/\sqrt{\tau}$. The open black circles were measured with a traditional clock using an uncorrelated input state. The dashed black line at $1.85 \times 10^{-9}\,\mathrm{s}^{1/2}/\sqrt{\tau}$ indicates the SQL at 100% signal contrast.

$\sigma(\tau) = 1.1 \times 10^{-9}\,\mathrm{s}^{1/2}/\sqrt{\tau}$, a factor of 2.8(3) in variance below the SQL [22]. At longer times we reach a noise floor at 10^{-10} fractional stability (0.7 Hz absolute stability) due to slow drifts of the magnetic field in our apparatus.

The performance of our clock only benefits from squeezing because we impose the constraint of a short Ramsey precession time. For longer interrogation times the classical noise dominates the initial phase noise and our clock is not projection-noise-limited. There are, however, realistic scenarios where external constraints or local oscillator dephasing limit the allowable Ramsey precession time and where metrological performance could be improved by the use of squeezed input states [13, 14]. For clocks with duty factors near unity [23], the greater sensitivity of a squeezed clock also enables the investigation of systematic effects in a shorter integration time.

If the classical frequency noise could be controlled at the level of $\sim 100\,\mu\mathrm{Hz}$, perhaps by operating in a magnetic trap at the magic bias field of 3.23 G, the squeezed lifetime could be extended sufficiently to allow a squeezed clock to operate with a Ramsey precession time of one second, as demonstrated by Treutlein et al. on a microchip similar to the one used in the present experiment [24]. Even without improvements to our squeezed-state preparation or 9 s cycle time, this could yield a short-term stability of $\sigma(\tau) \approx 2 \times 10^{-13}\,\mathrm{s}^{1/2}/\sqrt{\tau}$, competitive with the stability of current fountain clocks [25].

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